

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) CIVIL ENGINEERING

SEMESTER ONE EXAM 2022/2023

MATHEMATICAL METHODS FOR CIVIL ENGINEERING

MODULE NO: CIE4022

Date: Tuesday 10th January 2023

Time: 10:00 – 13:00

INSTRUCTIONS TO CANDIDATES:

This is an OPEN book examination

There are FOUR questions

Answer ALL Questions

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of
100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

Question 1

(a) Find $\frac{dy}{dx}$ when: $y = x^2 + x + 1 + \frac{1}{x} + \frac{1}{x^2}$ (2 marks)

Evaluate your answer at $x = 1$ and $x = -1$ (2 marks)

(b) Given that $y = \sin(3x) - \cos(3x)$ find $\frac{d^4y}{dx^4} = 81y$ (4 marks)

(c) Given that: $y = e^{2x} - \frac{1}{x}$ show that: $x^2 \frac{dy}{dx} - 1 = 2x^2 e^{2x}$ (4 marks)

(d)

(i) Expand the right hand side of: $y = (x - 2)^2(x + 3)$. (2 marks)

(ii) Sketch the graph. (2 marks)

(iii) Write down the co-ordinates any points of intersection with the axes. (3 marks)

(iv) Find the co-ordinates of any turning points and points of inflection. (6 marks)

Total Marks 25 Marks

Question 2

(a) Find $\frac{dy}{dx}$ when:

(i) $y = x^2 \sin(x)$ (3 marks)

(ii) $y = \frac{\sin(x)}{x^2}$ (3 marks)

(iii) $y = \sin(x^2)$ (3 marks)

(b) Show that: $\frac{1}{n} \frac{dy}{dx} + (nx + m) \sin(nx) = \cos(nx)$ given that: $y = (nx + m) \cos(nx)$ (5 marks)

(c) Differentiate $\frac{e^x \sin(x)}{x^2}$ (5 marks)

(d) The volume of a cylinder is given by: $V = \pi r^2 h$

(i) Suppose that $h = r + 1$, ($r > 0$) then find $\frac{dV}{dr}$ in terms of r . (2 marks)

(ii) Assuming that radius r can be measured with error $\Delta r = 0.01$, find an approximation to the % error in the volume when $r = 1$ and when $r = 2$. (4 marks)

Total Marks 25 Marks

School of Engineering
 BEng (Hons) Civil Engineering
 Semester One Exam - 2022/2023
 Mathematical Methods for Civil Engineering
 Module No: CIE4022

Question 3

(a) Integrate:

(i) $\int x^2 + x + 1 + \frac{1}{x} + \frac{1}{x^2} dx$ (3 marks)

(ii) $\int \frac{1}{\sqrt[3]{k^2}} dk$ (3 marks)

(iii) $\int \frac{20x-10}{x^2-x+10} dx$ (2 marks)

(v) $\int \sin(a) - \cos(2a) + e^{3a} da$ (3 marks)

(b) Find the exact value and an approximation to the definite integral:

$$\int_{-3}^2 (x - 2)^2(x + 3)dx$$

using the Trapezium Rule with 5 strips (6 marks)

(c)

(i) Evaluate the definite integrals:

$\int_{-1}^1 x - x^3 dx$, $\int_{-1}^0 x - x^3 dx$ and $\int_0^1 x - x^3 dx$ (3 marks)

(ii) Sketch the graph: $y = x - x^3$, for $-2 \leq x \leq 2$ (2 marks)

(d) Integrate:

$\int (3p - 1)^6 dp$ (3 marks)

Giving your answer in the form:

$$\frac{(3p - 1)^n}{m}$$

where n and m are integers.

Total Marks 25 Marks

PLEASE TURN THE PAGE...

School of Engineering
 BEng (Hons) Civil Engineering
 Semester One Exam - 2022/2023
 Mathematical Methods for Civil Engineering
 Module No: CIE4022

Question 4

- (a) (i) Sketch the graph of $y = \sqrt{9 - x^2}$ $0 \leq x \leq 3$ (2 marks)

(ii) Integrate:

$$\int_0^3 \sqrt{9 - x^2} \, dx$$

using the substitution: $x = 3 \sin(\theta)$ and the identity: $\cos^2(\theta) = \frac{1}{2}[1 + \cos(2\theta)]$.

(6 marks)

Hence, find the area of a circle with radius $r = 3$.

(2 marks)

- (b) Integrate:

(i) $\int x^2 e^x dx$ (6 marks)

(ii) $\int 3x^2 (x^3 - 2)^{-2/3} dx$ (4 marks)

- (c) A function that passes through the point (0,5) is differentiated to produce $\frac{dy}{dx} = 2e^x + 2$.
 Find the original function. (2 marks)

- (d) Evaluate: $\int_{0.01}^1 \ln(x) dx$ (2 marks)

Comment on the value of $\int_a^1 \ln(x) dx$ as 'a' tends to zero. (1 marks)

Total Marks 25 Marks

END OF QUESTIONS

FORMULA SHEET FOLLOWS OVER THE PAGE....

PLEASE TURN THE PAGE...

School of Engineering
 BEng (Hons) Civil Engineering
 Semester One Exam - 2022/2023
 Mathematical Methods for Civil Engineering
 Module No: CIE4022

FORMULA SHEET

Function $f(x)$ or y	Differentiation $f'(x)$ or $\frac{dy}{dx}$
x^n	nx^{n-1}
e^x	e^x
e^{ax}	ae^{ax}
$\ln(x)$	$\frac{1}{x}$
$\sin(x)$	$\cos(x)$
$\sin(ax)$	$a \cos(ax)$
$\cos(ax)$	$-a \sin(ax)$

PLEASE TURN THE PAGE...

School of Engineering
 BEng (Hons) Civil Engineering
 Semester One Exam - 2022/2023
 Mathematical Methods for Civil Engineering
 Module No: CIE4022

	<u>Chain rule</u>	<u>Product rule</u>	<u>Quotient rule</u>
<u>Differentiation</u>	$y = f(g(x)) \quad u = g(x)$ $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$	$y = u v$ $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$	$y = \frac{u}{v}$ $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
	<u>By parts</u>		
<u>Integration</u>	$y = u \frac{dv}{dx}$ $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$		

END OF PAPER