ENG151

UNIVERSITY OF GREATER MANCHESTER

SCHOOL OF ENGINEERING AND BUILT ENVIRONMENT

B.ENG (HONS) MECHANICAL ENGINEERING

SEMESTER ONE EXAMINATION 2025/2026

MECHANICS OF MATERIALS AND MACHINES

MODULE NO: AME5012

Date: Friday 16th January 2026 **Time: 10am – 12 noon**

INSTRUCTIONS TO CANDIDATES:

There are FIVE questions.

Answer FOUR questions.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE:

Formula Sheets (attached after questions).

Q1. Thick and Thin Pressure Vessel Design for Offshore Hydrogen Storage

A closed-ended cylindrical **hydrogen storage tank** is used in an offshore renewable-energy platform.The vessel is made of high-strength steel and is subjected to both **internal** and **external** pressures.The internal pressure is **10 MPa**, and the surrounding seawater exerts an external pressure of **4 MPa**.

The internal diameter of the vessel is **500 mm**, and the external diameter is **800 mm**.The material has a **Young's modulus of 205 GPa**, a **Poisson's ratio of 0.30**, and a **yield strength of 360 MPa**. Assume the vessel behaves elastically, remains within the linear range, and has **closed ends**.

Tasks

a) Using **Lame's equations**, calculate the **radial** and **hoop stresses** at the **inner** and **outer surfaces** of the cylinder when subjected to the given internal and external pressures.

(8 marks)

b) Determine the **maximum shear stress** in the cylinder wall and state **clearly where it occurs**, explaining your reasoning with reference to the stress distribution.

(4 marks)

c) For the closed-ended vessel, calculate the **longitudinal (axial) stress**, then determine the **hoop strain** at both the **inner** and **outer surfaces**, using the given material properties.

(6 marks)

d) The design team plans to **redesign the vessel as a thin-walled storage tank** for an on-shore hydrogen system operating at an internal pressure of **10 MPa** with negligible external pressure.
If the **design factor** is **3** and the yield strength of the steel is **360 MPa**, determine:

- the **permissible working stress**,
- the **minimum wall thickness** for an inner radius of 250 mm, using the thin-wall relation $\sigma_h = \frac{pr}{t}$, and
- the resulting radius-to-thickness ratio, commenting on whether the thin-wall assumption ($t \leq d/20$) is satisfied.

(7 marks)

(Total 25 marks)

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Q2. Struts - UAV Landing-Gear Leg (Buckling & Design)

A medium-size quadcopter uses a lightweight **aluminium landing-gear leg** that can be idealised as a slender strut. One end is **rigidly clamped to the fuselage**, and the ground-contact end behaves approximately **pinned** during touchdown. Assume linear-elastic behaviour and small deflections.

Given:

- Length $L = 0.60\ m$
- Hollow circular tube: outer diameter $D_0 = 22\ mm$, wall thickness $t = 1.5\ mm$
- End condition: **fixed–pinned** (use the appropriate effective-length factor from the equation sheet)
- Material: Aluminium alloy, $E = 72\ GPa$, yield stress $\sigma_y = 505\ MPa$
- Rankine constant $a = 1/9000$

Tasks

a) Determine the **cross-sectional area**, **second moment of area**, **radius of gyration**, and the **slenderness ratio** of the landing-gear leg.

(8 marks)

b) Using the **limiting slenderness criterion** provided on the equation sheet, decide whether **Euler elastic buckling** is applicable for this member. Clearly state and justify your conclusion.

(4 marks)

c) Compute the **Euler critical load** for buckling of this fixed–pinned member.

(5 marks)

d) Using the **Rankine–Gordon** approach, calculate the **maximum load** the leg can carry. Then recommend a **safe working load** using a **design factor of 2.5**, and briefly justify which model (Euler or Rankine) should govern the design for this UAV component.

(8 marks)

(Total 25 marks)

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Q3: Principal Stresses, Mohr's Circle & Safety - EV Suspension Control Arm

A forged aluminium **suspension control arm** in an electric vehicle is analysed under a peak cornering load. At a fillet hotspot, the **plane-stress** components (MPa) are:

$$\sigma_{xx} = 85, \sigma_{yy} = -55, \tau_{xy} = 40,$$

with $\sigma_{zz} = 0$,zz=0, $\tau_{yz} = \tau_{zx} = 0$.

Material: high-strength Al alloy, $E = 70\,GPa$, yield strength $\sigma_y = 450\,MPa$.

Use the standard formulae provided on your equation sheet. Show a clear, labelled construction where requested.

Tasks

a) Sketch the **elemental stress block** showing the given σ_{xx}, σ_{yy}, τ_{xy}, with correct sign convention and directions.

(4 marks)

b) Using the **matrix (eigenvalue) method**, form the characteristic equation of the 2D stress tensor and compute the **principal stresses** σ_1, σ_2.

(7 marks)

c) Using **Mohr's circle**, determine:
- the **principal stresses** σ_1, σ_2 and the **maximum in-plane shear stress** τ_{max};
- the **principal plane orientation** (angle measured counter-clockwise from the x-axis).

Provide a neat Mohr's circle **construction** (centres, radii, key points, and angle mapping).

(9 marks)

d) Evaluate the **von Mises equivalent stress** at this point and hence the **factor of safety** against yielding of the control arm. Comment (one sentence) on whether yielding is likely under this load case.

(5 marks)

(Total 25 marks)

Q4: Application of Beam Theory - Robotic Arm Segment

A section of a **robotic manipulator arm** can be idealised as a hollow circular cross-section, made of carbon-steel alloy, carrying its own weight and a **uniformly distributed payload load (Figure Q4)**.

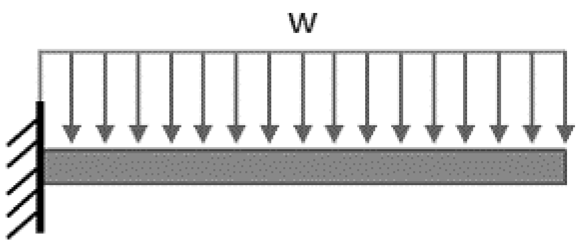

Figure Q4: robotic manipulator arm.

Given:

- Length $L = 0.8\, m$
- Outer diameter $D_0 = 40\, mm$
- Wall thickness $t = 3\, mm$
- Uniformly distributed load $w = 800\, N/m$
- Young's modulus $E = 210\, GPa$
- Yield strength $\sigma_y = 400\, MPa$

Tasks

a) Derive the **shear force** and **bending moment** equations for the cantilever, and determine their **maximum values**.

(6 marks)

b) Compute the **second moment of area** for the hollow circular section, and determine the **maximum bending stress**.

(6 marks)

c) Calculate the **maximum deflection** due to the uniformly distributed load.

(6 marks)

Question 4 continued on page 6

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Question 4 continued

d) Using a **design factor of 2.5**, assess whether the beam is safe for operation under these conditions.

(4 marks)

e) If the deflection exceeds the acceptable limit, suggest **two feasible design strategies** (geometrical or material-based) to enhance stiffness without excessive weight increase.

(3 marks)

(Total 25 marks)

Q5. Application of Beam Theory - Overhead Crane Runway Beam

An **overhead gantry crane** uses a **beam** as a runway girder to carry the moving hoist and trolley.
When fully loaded, the beam experiences an **effective uniformly distributed load** that includes its self-weight and the trolley load **(Figure Q5).**

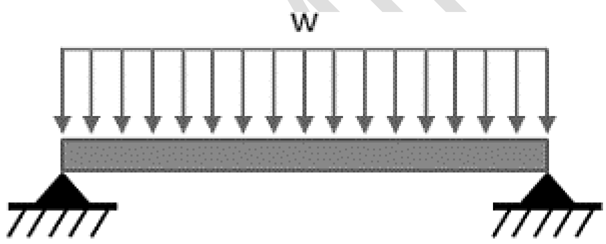

Figure Q5: Simply supported beam-shelving system.

Question 5 continued on page 7

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Question 5 continued

Given:

- Span $L = 3.0\,m$
- Equivalent UDL $w = 6000\,N/$
- Cross-section approximated as a **rectangular section** of width $b = 80\,mm,$ height $h = 180\,mm$
- Steel modulus $E = 210\,GPa$
- Yield stress $\sigma_y = 360\,MPa$

Tasks

a) Derive and sketch the **shear-force** and **bending-moment** diagrams for the beam under UDL, stating the **maximum values**.

(6 marks)

b) Determine the **maximum bending stress** and specify whether it occurs on the top or bottom surface.

(6 marks)

c) Calculate the **mid-span deflection** of the beam under full load.

(6 marks)

d) Using a **factor of safety of 2**, decide whether the beam is safe in service.

(4 marks)

e) Briefly discuss **one modification** (e.g. section shape or material change) that would reduce deflection while maintaining similar mass.

(3 marks)

(**Total 25 marks)**

END OF QUESTIONS

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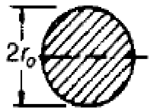
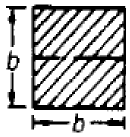
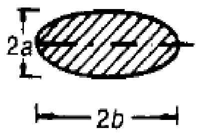
FORMULA SHEET

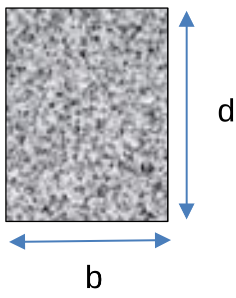
Deflection:

$$M_{xx} = EI \frac{d^2y}{dx^2}$$

Section Shape	$A(m^2)$	$I_{xx}(m^4)$
	πr^2	$\frac{\pi}{4} r^4$
	b^2	$\frac{b^4}{12}$
	πab	$\frac{\pi}{4} a^3 b$

For solid rectangular Cross-section

$$I_{xx} = \frac{bd^3}{12}$$



Plane Stress:

a) Stresses in function of the angle Θ:

$$\sigma_x(\theta) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_y(\theta) = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau_{xy}(\theta) = -\frac{\sigma_x - \sigma_y}{2} sin(2\theta) + \tau_{xy} cos(2\theta)$$

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b) Principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

Hoop Stress of a thin Pressure Vessel:

$$\sigma_{h} = \frac{\Delta P r}{t}$$

Lame's equation

The equations are known as "Lame's Equations" for radial and hoop stress at any specified point on the cylinder wall.   Note: R_1 = inner cylinder radius, R_2 = outer cylinder radius

$$\sigma_C = a + \frac{b}{r^2}$$

$$\sigma_R = a - \frac{b}{r^2}$$

The corresponding strains format is:

$$\varepsilon_c = 1/E\ \{\sigma_c - \nu(\sigma_r + \sigma_L)\}$$

$$\varepsilon_r = 1/E\ \{\sigma_r - \nu(\sigma_c + \sigma_L)\}$$

$$\varepsilon_L = 1/E\ \{\sigma_L - \nu(\sigma_c + \sigma_r)\}$$

$$\sigma_L = \frac{P_1 R_1^2 - P_2 R_2^2}{(R_2^2 - R_1^2)}$$

$$\tau_{max} = \frac{\sigma_c - \sigma_r}{2} = \frac{b}{r^2}$$

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Stress

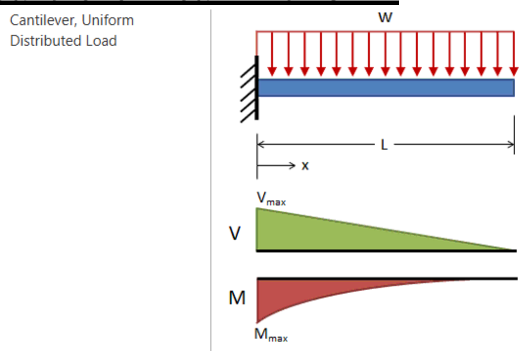
σ = Force/Area = F/A

Hook's law

$\sigma = E \cdot \varepsilon$

$\varepsilon = \Delta L / L$

Cantilever Beam with UDL:



Deflection:

$$\delta = -\frac{wx^2}{24EI}(6L^2 - 4Lx + x^2)$$

$$\delta_{max} = \frac{wL^4}{8EI} \quad @\, x = L$$

Slope:

$$\theta = -\frac{wx}{6EI}(3L^2 - 3Lx + x^2)$$

$$\theta_{max} = \frac{wL^3}{6EI} \quad @\, x = L$$

Shear:

$$V = +w\,(L - x)$$

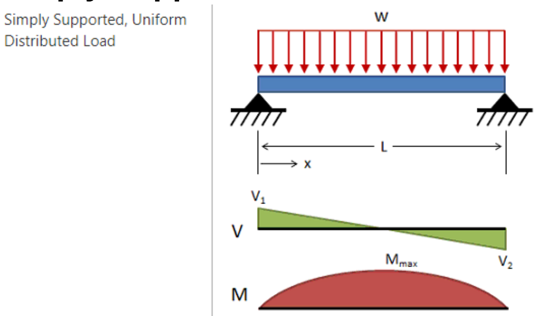
$$V_{max} = +wL \quad @\, x = 0$$

Moment:

$$M = -w\,(L - x)^2 / 2$$

$$M_{max} = -wL^2 / 2 \quad @\, x = 0$$

Simply Supported Beam with UDL:



Deflection:

$$\delta = -\frac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$\delta_{max} = \frac{5wL^4}{384EI} \quad @\, x = L/2$$

Slope:

$$\theta = -\frac{w}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$\theta_1 = -\frac{wL^3}{24EI} \quad @\, x = 0$$

$$\theta_2 = +\frac{wL^3}{24EI} \quad @\, x = L$$

Shear:

$$V = w\,(L/2 - x)$$

$$V_1 = +wL / 2 \quad @\, x = 0$$

$$V_2 = -wL / 2 \quad @\, x = L$$

Moment:

$$M_{max} = wL^2 / 8 \quad @\, x = L/2$$

Maximum Bending Stress

$$\sigma_{max} = \frac{M_{max} y}{I}$$

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Yield Criterion

Von Mises

$$\sigma_{von\ Mises} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

Quadratic equation: $ax^2+bx+c=0$

Solution:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Safety factor and Design factor:

$$\textbf{Factor Of Safety} = \frac{\sigma_{yield}}{\sigma_{applied}}$$

$$\textbf{Design Factor} = \frac{\textbf{Design Stress}}{\textbf{Allowable Stress}}$$

Struts:

$$I = k^2 A$$

$$k = \sqrt{\frac{I}{A}}$$

Eigenvalues

$$|A - \lambda \mathrm{I}| = 0$$

Eigenvectors

$$(A - \lambda_r \mathrm{I})x_r = 0$$

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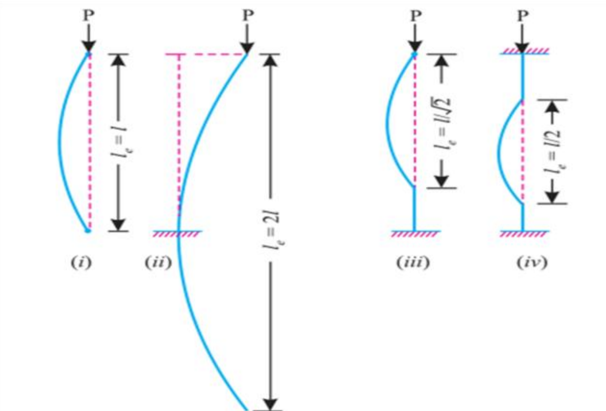
Euler validity

$$Slenderness\ ratio = SR = \frac{L_e}{k} \geq \pi \sqrt{\frac{E}{\sigma_{yield}}}$$



(i) Both ends pin jointed or hinged or rounded or free.
(ii) One end fixed and other end free.
(iii) One end fixed and the other pin jointed.
(iv) Both ends fixed.

$$a = \frac{\sigma_c}{\pi^2 \cdot E}$$

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