ENG152

UNIVERSITY OF GREATER MANCHESTER

SCHOOL OF ENGINEERING AND BUILT ENVIRONMENT

BEng (HONS) MECHANICAL, ELECTRICAL & ELECTRONIC ENGINEERING

SEMESTER ONE EXAMINATION 2025/2026

ENGINEERING MODELLING AND ANALYSIS

MODULE NO: AME5014

Date: Tuesday 13th January 2026 **Time: 10am – 12 noon**

INSTRUCTIONS TO CANDIDATES:

There are EIGHT questions.

Answer ANY FIVE questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used if data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE:

Formula Sheets (attached following questions).

School of Engineering and Built Environment
BEng (Hons) Mechanical, Electrical and Electronic Engineering
Semester One Examination 2025/2026
Engineering Modelling and Analysis
Module No. AME5014

Q1: Differentiation and Integration

(a) Dynamic Electrical System – Damped Oscillation

In a **damped RLC circuit**, the instantaneous charge on the capacitor varies with time as:

$$Q(t) = 5e^{-2t}cos\,(6t)$$

where Q is in coulombs (C) and t is in seconds (s).

The **current** $i(t)$ (in amperes, A) is the rate of change of charge:

$$i(t) = \frac{dQ}{dt}$$

Tasks:

i. Derive an expression for $i(t)$ using appropriate differentiation rules.
ii. Determine the **maximum current** and the **time** at which it first occurs.
iii. Briefly comment on the physical effect of the exponential term in this system.

(10 Marks)

(b) Mechanical Power Transmission – Variable Torque System

A variable-speed rotating shaft experiences a torque given by:

$$\tau(\theta) = 4\sin(2\theta) + 2\theta$$

where τ is in Nm and θ is the angular displacement in radians.

Tasks:

i. Determine the **work done** (in J) by the torque as the shaft rotates from $\theta = 0$ to $\theta = \pi$.
ii. If the shaft completes this rotation in $0.5\ s$, calculate the **average power** transmitted.
iii. Discuss how the sinusoidal torque term relates to *fluctuating load conditions* in real machinery (e.g. reciprocating engines or wind turbines).

(10 Marks)

Total 20 Marks

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Q2: Second Order Differential Equation

Solve <u>ONE</u> of the <u>TWO</u> parts below:

<u>Part 1: Overdamped Vehicle Suspension System</u>

(a) A vehicle suspension shock absorber is modelled by

$$m\,x''(t) + c\,x'(t) + k\,x(t) = 0,$$

where $x(t)$ is the vertical displacement (m) and t is time (s).
Given:

$$m = 2\;kg, c = 25Ns/m, k = 40\;N/m,$$

with initial conditions

$$x(0) = 0.06\;m, x'(0) = -0.12\;m/s.$$

Tasks:
i. Form the **characteristic equation**, compute its **discriminant**, and find the **two roots**.
ii. Write the **general solution** and use the **initial conditions** to determine the constants; hence give $x(t)$.
iii. Using **only the discriminant and the root types**, classify the response (underdamped/critical/overdamped) and briefly describe the qualitative motion.

(14 Marks)

(b) Using your result from part (a), evaluate the displacement at

$$t = 0.5\;s,\;\;1.0\;s,\;\;1.5\;s.$$

Tasks:
i. Write the **explicit expression** for $x(t)$ with the constants you obtained in part (a).
ii. **Evaluate** $x(t)$ at $t = 0.5$, 1.0, $1.5s$ (show all working) and present the results in a **table** (columns: t in s, $x(t)$ in m.

Q2 continued over the page

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Q2 continued

iii. Briefly **comment** on the time response you observe (e.g., monotonic decay, absence of oscillation), linking your observation to the **root types** from part (a).

(6 Marks)

Total 20 Marks

Part 2: DC Motor Electrical Dynamics

a) When a **DC motor** is suddenly de-energised, the current through its inductance and resistance interacts with the motor's back-EMF circuit. The current $i(t)$ (A) is governed by

$$Li''(t) + R_1 i'(t) + R_2 i(t) = 0,$$

where L is the combined inductance of the armature and supply line, R_1 represents total series resistance, and R_2 reflects additional damping due to the back-EMF coupling.

Given:

$$L = 0.2\ H, R1 = 8\ \Omega, R2 = 12\ \Omega,$$

and initial conditions

$$i(0) = 6.0\ A, i'(0) = \frac{0\ A}{s}.$$

Tasks:
i. Divide through by L to obtain the **standard form** of the equation.
ii. Form the **characteristic equation**, compute its **discriminant**, and find the **two roots**.
iii. Write the **general solution**, apply the **initial conditions** to determine constants, and give the full expression for $i(t)$.
iv. Using **only the discriminant and the root types**, classify the transient (underdamped / critically / overdamped) and briefly describe the current behaviour.

(14 Marks)

Q2 continued over the page

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Q2 continued

(b) Using your analytical solution from part (a), calculate and tabulate the current $i(t)$ at

$$t = 0.05\ s,\ 0.10\ s,\ 0.15\ s.$$

Tasks:
i. Write the **explicit expression** for $i(t)$ with constants substituted from part (a).
ii. **Evaluate** $i(t)$ at the given times (show working) and present results in a **table** (columns: t/s, $i(t)/A$).
iii. Briefly **comment** on the transient response and relate the behaviour to the **root types** found earlier.

(6 Marks)

Total 20 Marks

Q3: First Order Differential Equation

Solve <u>ONE</u> of the <u>TWO</u> parts below:

<u>Part 1: Falling Body with Air Resistance</u>

(a) A small object of **mass** $m = 2.0\ kg$ is dropped from rest and falls vertically under gravity in air. The air resistance force is **proportional to velocity**, with a proportionality constant $b = 2.0\ Ns/m$.

The motion of the object is governed by:

$$m\,\frac{dv}{dt} = -mg + b\,v(t),$$

where

- $v(t)$ is the velocity (m/s) at time $t\ (s)$,
- $g = 9.81\ m/s^2$ is gravitational acceleration.

Q3 continued over the page

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Q3 continued

Tasks:
i. Rearrange the differential equation into the **standard first-order form** and solve for $v(t)$ using the **method of separation of variables** (or integrating factor).
ii. Apply the initial condition $v(0) = 0$ to obtain the **complete solution**.
iii. Identify the **terminal velocity** (at $t = 60\, minutes$) of the object from your expression and explain its physical meaning.

(12 Marks)

(b) Using your solution from part (a), determine the **velocity of the object** at

$t = 1.0\ s$ and $t = 2.0\ s$.

Tasks:
i. Write the **explicit expression** for $v(t)$ obtained in part (a).
ii. **Evaluate** $v(t)$ at $t = 1.0\ s$ and $t = 2.0\ s$ (show all working).
iii. Briefly **comment** on how the velocity changes over time and how it compares to the terminal velocity.

(8 Marks)

Total 20 Marks

Q3 continued over the page

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Q3 continued

<u>Part 2:</u>

(a) When a DC source is suddenly **disconnected** from a series **RL circuit**, the current flowing in the inductor decays exponentially due to the inductor's stored energy.
The circuit is governed by

$$L\,\frac{di(t)}{dt} + R\,i(t) = 0,$$

where

- $i(t)$ = current (A) at time t (s),
- L = inductance (H),
- R = resistance (Ω).

Given:

$$L = 0.5\ H, R = 20\ \Omega,$$

and initial current

$$i(0) = 4.0\ A.$$

Tasks:
i. Rearrange the differential equation into **standard first-order form** and solve for $i(t)$ using the **method of separation of variables** (or integrating factor).
ii. Apply the initial condition $i(0) = 4.0A$ to obtain the **complete solution**.
iii. Determine the **time constant,** $\tau = {}^{L}/_{R}$ of the circuit and explain its physical meaning.

(12 Marks)

Q3 continued over the page

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Q3 continued

(b) Using your solution from part (a), determine the **current in the inductor** at

$t = 0.05\ s$ and $t = 0.1\ 0\ s$.

Tasks:
i. Write the **explicit expression** for $i(t)$ obtained in part (a).
ii. **Evaluate** $i(t)i$ at $t = 0.05\ s$ and $t = 0.10\ s$ (show all working).
iii. Briefly **comment** on how the current changes with time and relate it to the circuit's time constant.

(8 Marks)

Total 20 Marks

Q4: Laplace Transforms

Solve ONE of the TWO parts below:

Part 1: Translational Drive with Viscous Drag

(a) A cart of mass $m = 4\,kg$ experiences viscous drag $b = 8\,Ns/m$.
At $t = 0\ s$, a constant driving force of $F_0 = 40\ N$ is suddenly applied and held (i.e. a **step input**).
The velocity $v(t)$ satisfies:

$$m\frac{dv}{dt} + b\,v = F_0\,u(t), v(0) = 0\ m/s,$$

where $u(t)$ is the unit step function.

Using **Laplace transforms**, derive an explicit expression for $v(t)$.

(12 Marks)

Q4 continued over the page

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Q4 continued

(b) Using your result from part (a):

i. Determine the time t (in seconds) when the velocity reaches $4.0\,m/s$.

ii. Comment briefly on how the velocity changes with time and what happens after a long period.

(8 Marks)

Total 20 Marks

<u>Part 2: Transient Response of a RL Circuit</u>

(a) A series RL circuit has $L = 0.20\,H$and $R = 5.0$.
At $t = 0\,s$, a DC source of $V_s = 10\,V$ is connected (**step input**).
The current $i(t)$ satisfies:

$$L\frac{di}{dt} + R\,i = V_s\,u(t), i(0) = 0\,A.$$

Using **Laplace transforms**, derive an explicit expression for $i(t)$.

(12 Marks)

(b) Using your result from part (a):

i. Determine the time t (in seconds) when the current reaches $1.5\,A$.

ii. Comment briefly on how the current changes with time and what happens after a long period.

(8 Marks)

Total 20 Marks

Q5: Fourier transform- Frequency Analysis of an Excitation Signal

A **mechatronic actuation system** applies a **square-pulse excitation** f(t)f(t)f(t) to an energy-storage element (e.g., a **force pulse** on a mechanical system or a **voltage pulse** in an electrical circuit). The signal is

$$f(t) == \begin{cases} 5, & -3 \le t \le 0, \\ -5, & 0 \le t \le 3, \\ 0, & |t| > 3. \end{cases}$$

(a) Sketch the waveform of $f(t)$ and **describe its key features** (amplitude, duration, symmetry, discontinuities).

(6 Marks)

(b) Derive the **Fourier transform** $F(\omega)$ of the signal analytically.
Then, using your result, **explain in simple terms** how the size of $F(\omega)$ changes with frequency ω (i.e., which ranges are stronger or weaker).

(14 Marks)

Total 20 Marks

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Q6: Matrices - Drone Altitude Control Stability

A **quadcopter drone** is hovering at a fixed altitude. To maintain altitude, the controller adjusts the rotor thrust based on position error e (difference from target height) and its rate $\dot{e}$ (vertical speed).

The linearised **altitude error dynamics** near steady hover are given by:

$$\dot{x} = A\,x,$$
$$x = \begin{bmatrix} e \\ \dot{e} \end{bmatrix},$$
$$A = \begin{bmatrix} 0 & 1 \\ -16 & -10 \end{bmatrix}.$$

Here e is the altitude error (m), and $\dot{e}$ is the vertical velocity (m/s).

(a) Determine the eigenvalues of matrix A.
Based on the values obtained, clearly state whether the **drone's altitude control system** is **stable or unstable**, and briefly explain why.

Clue:
If all eigenvalues are **negative real numbers**, the altitude error **decays with time** - the system is **stable**.
If any eigenvalue is **positive**, the altitude error **grows with time** - the system is **unstable**.

(10 Marks)

(b) Find **one eigenvector** corresponding to **each** eigenvalue of A.

(10 Marks)

Total 20 Marks

Q7: Simpson's rule - Robotic Arm Motion Analysis

In a robotic pick-and-place system, a **servo-driven arm joint** moves along a programmed trajectory.
The **angular velocity**, $\omega(t)$, of the joint motor is recorded over a short operation cycle, as shown below:

Time, t (s)	0	0.5	1.0	1.5	2.0	2.5	3
Angular velocity, $\omega(t)$ (rad/s)	0	4.5	8.0	10.2	11.5	12.0	12.2

(a) Sketch the graph of **angular velocity** $\omega(t)$ versus **time** t using the data provided. Label the axes clearly and annotate the graph to indicate how the angular velocity changes with time.

(6 Marks)

(b) Using **Simpson's Rule**, estimate the **total angular displacement** (in radians) of the robotic joint between $t = 0\ s$ and $t\ =\ 3\ s$.

Briefly comment on the physical meaning of your result in the context of **robot motion** (e.g., how far the joint has rotated).

(14 Marks)

Total 20 Marks

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Q8: Partial derivative and double integrals

(a) Capacitor Heating
In a high-power circuit, the **temperature** T (in $°C$) of a capacitor surface depends on both the **voltage** v (in V) applied across it and **time** t (in s).
The temperature variation is described by:

$$T(v,t) = e^{-0.2t} * \sin(2v)$$

The rate of change of temperature is represented by:

$$Z = \frac{\partial T}{\partial v} + \frac{\partial^2 T}{\partial v \partial t}$$

Evaluate Z in ${}^{\circ}C/_{Vs}$ where $x = \frac{\pi}{6}$ and $t = 3\ s$.

(10 Marks)

(b) Torque Distribution on a Robotic Arm Base

The **torque density** $f(x,y)$ (in $N \cdot m/m^2$) on a square robotic base is distributed according to the function:

$$f(x,y) = 6\,x^2 - 4\,y^3 + 2.$$

The total torque acting on the base is given by:

$$f(x,y) = \int_{x=0}^{x=5} \int_{y=0}^{y=3} f(x,y)\,dy\,dx.$$

Evaluate the total torque T in Nm and state its physical meaning in this context.

(10 Marks)

Total 20 Marks

END OF QUESTIONS

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FORMULA SHEET

Partial Fractions

$$\frac{F(x)}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$$

$$\frac{F(x)}{(x+a)(x+b)^2} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+b)^2}$$

$$\frac{F(x)}{(x^2+a)} = \frac{Ax+B}{(x^2+a)}$$

Small Changes

z = f(u, v, w)

$$\delta z \simeq \frac{\partial z}{\partial u}.\delta u + \frac{\partial z}{\partial v}.\delta v + \frac{\partial z}{\partial w}.\delta w$$

Total Differential

z = f(u, v, w)

$$dz = \frac{\partial z}{\partial u}du + \frac{\partial z}{\partial v}dv + \frac{\partial z}{\partial w}dw$$

Rate of Change

z = f(u, v, w)

$$\frac{dz}{dt} = \frac{\partial z}{\partial u}.\frac{du}{dt} + \frac{\partial z}{\partial v}.\frac{dv}{dt} + \frac{\partial z}{\partial w}.\frac{dw}{dt}$$

Eigenvalues

$$|A - \lambda \mathrm{I}| = 0$$

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Formula sheet continued

Eigenvectors

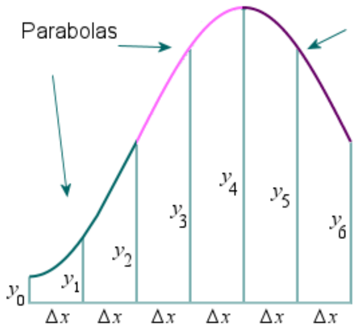
$$(A - \lambda_r \mathrm{I})x_r = 0$$

Integration

$$\int u.\frac{dv}{dx}dx = uv - \int v.\frac{du}{dx}dx$$

Simpson's rule

To calculate the area under the curve which is the integral of the function **Simpson's Rule** is used as shown in the figure below:



The area into *n* equal segments of width *Δx*. Note that in Simpson's Rule, *n* must be EVEN. The approximate area is given by the following rule:

$$Area = \int_a^b f(x)dx = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 \ldots + 4y_{n-1} + y_n)$$

Where $\Delta x = \frac{b-a}{n}$

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Differential equation

Homogeneous form:

$$a\ddot{y} + b\dot{y} + cy = 0$$

Characteristic equation:

$$a\lambda^2 + b\lambda + c = 0$$

Quadratic solutions :

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

i. If $b^2 - 4ac > 0$, λ_1 and λ_2 are distinct real numbers then the general solution of the differential equation is:

$$y(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

A and B are constants.

ii. If $b^2 - 4ac = 0$, $\lambda_1 = \lambda_2 = \lambda$ then the general solution of the differential equation is:

$$y(t) = e^{\lambda t}(A + Bx)$$

A and B are constants.

iii. If $b^2 - 4ac < 0$, λ_1 and λ_2 are complex numbers then the general solution of the differential equation is:

$$y(t) = e^{\alpha t}[Acos(\beta t) + Bsin(\beta t)]$$

$$\alpha = \frac{-b}{2a} \quad and \quad \beta = \frac{\sqrt{b^2 - 4ac}}{2a}$$

A and B are constants.

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Inverse of 2x2 matrices:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The inverse of A can be found using the formula:

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Modelling growth and decay of engineering problem

$C(t) = C_0 e^{kt}$

$k > 0$ gives exponential growth

$k < 0$ gives exponential decay

First order system

$$y(t) = k(1 - e^{-\frac{t}{\tau}})$$

Transfer function: $\dfrac{k}{\tau s+1}$

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Derivatives table:

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
k, any constant	0
x	1
x^2	$2x$
x^3	$3x^2$
x^n, any constant n	nx^{n-1}
e^x	e^x
e^{kx}	ke^{kx}
$\ln x = \log_e x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\sin kx$	$k \cos kx$
$\cos x$	$-\sin x$
$\cos kx$	$-k \sin kx$
$\tan x = \frac{\sin x}{\cos x}$	$\sec^2 x$
$\tan kx$	$k \sec^2 kx$
$\operatorname{cosec} x = \frac{1}{\sin x}$	$-\operatorname{cosec} x \cot x$
$\sec x = \frac{1}{\cos x}$	$\sec x \tan x$
$\cot x = \frac{\cos x}{\sin x}$	$-\operatorname{cosec}^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$

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Formula sheet continued

Integral table:

$f(x)$	$\int f(x)\,\mathrm{d}x$
k, any constant	$kx + c$
x	$\frac{x^2}{2} + c$
x^2	$\frac{x^3}{3} + c$
x^n	$\frac{x^{n+1}}{n+1} + c$
$x^{-1} = \frac{1}{x}$	$\ln\lvert x\rvert + c$
e^x	$\mathrm{e}^x + c$
e^{kx}	$\frac{1}{k}\mathrm{e}^{kx} + c$
$\cos x$	$\sin x + c$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin x$	$-\cos x + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
$\tan x$	$\ln(\sec x) + c$
$\sec x$	$\ln(\sec x + \tan x) +$
$\mathrm{cosec}\, x$	$\ln(\mathrm{cosec}\, x - \cot x) +$
$\cot x$	$\ln(\sin x) + c$
$\cosh x$	$\sinh x + c$
$\sinh x$	$\cosh x + c$
$\tanh x$	$\ln\cosh x + c$
$\coth x$	$\ln\sinh x + c$
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a} + c$

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<u>Laplace table:</u>

$f(t)$	$F(s)$		$f(t)$	$F(s)$
1	$\frac{1}{s}$		$u_c(t)$	$\frac{e^{-cs}}{s}$
t	$\frac{1}{s^2}$		$\delta(t)$	1
t^n	$\frac{n!}{s^{n+1}}$		$\delta(t-c)$	e^{-cs}
e^{at}	$\frac{1}{s-a}$		$f'(t)$	$sF(s)-f(0)$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$		$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos bt$	$\frac{s}{s^2+b^2}$		$(-t)^n f(t)$	$F^{(n)}(s)$
$\sin bt$	$\frac{b}{s^2+b^2}$		$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$		$e^{ct}f(t)$	$F(s-c)$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$		$\delta(t-c)f(t)$	$e^{-cs}f(c)$

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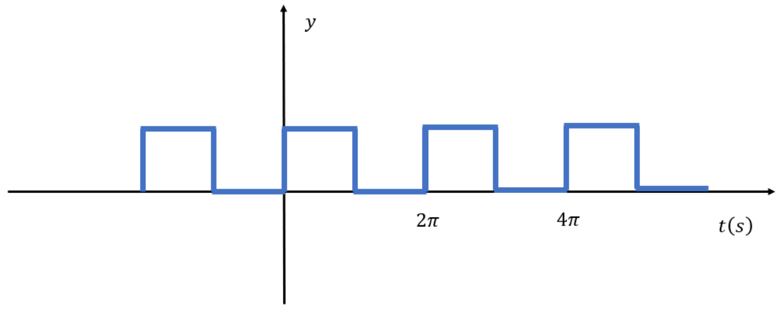
Fourier Series

The periodic square wave with Fourier Series and the coefficients of the Fourier Series



The function which represent the periodic square wave can be represented by

$$y = f(t)$$

Period of the function:

$$T = 2\pi \,\frac{sec}{cycle}$$

Fourier series of the function:

$$f(t) = a_0 + a_1 \cos(t) + a_2 \cos(2t) + a_3 \cos(3t) + \cdots + a_n \cos(nt)$$
$$+ b_1 \sin(t) + b_2 \sin(2t) + b_3 \sin(3t) + \cdots b_n \sin(nt)$$

Where, $n = 1,2,3,4,5, \ldots \ldots ..$

Alternatively,

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nt) + b_n \sin(nt))$$

Fourier Coefficients:

$$a_0 = \frac{1}{T}\int_0^T f(t)dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(nt)\,dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(nt)\,dt$$

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Useful Equations for Fourier transform

Fourier transform equation

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt$$

Inverse Fourier transform equation

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t} d\omega$$

Euler's formula for trigonometric identities

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

Where, $j = \sqrt{-1}$

For any arbitrary function

$$\int_a^b f(t)\delta(t - t_0)\, dt = f(t_0)$$

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