ENG167

UNIVERSITY OF GREATER MANCHESTER

SCHOOL OF ENGINEERING AND BUILT ENVIRONMENT

BEng (Hons) MECHANICAL ENGINEERING

SEMESTER ONE EXAMINATION 2025/26

ADVANCED THERMOFLUIDS & CONTROL

MODULE NO: AME6015

Date: Wednesday 14th January 2026 **Time: 2pm – 4pm**

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

Formula sheets follow after questions.

QUESTION 1

You are a mechanical engineer responsible for safety and ventilation analysis in a hydrogen energy testing facility. During a containment failure, **an explosion releases 3 kg of hydrogen–methane gas mixture** into a closed test room with internal dimensions of 33 m × 7 m × 4 m **(Figure 1)**. The room air is assumed to be **well-mixed**, and the ventilation system operates through an aperture of 6 m² with an average air velocity of 0.6 m s^{-1}**.**

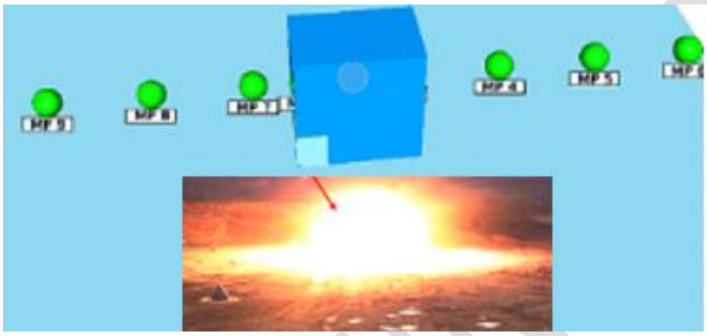

Figure 1 Explosion of Hydrogen-Methane Gas

Assuming steady ventilation and uniform mixing throughout the room and taking the density of air as 1.2 kg·m^{-3}:

a. Determine the initial concentration of gas (in ppm by mass) immediately after the release.

[11 Marks]

b. Estimate the time required for the gas concentration to decrease to a safe level of 1 ppm under continuous ventilation.

[14 Marks]

Show all necessary steps, equations, and assumptions used in your calculations.

Total 25 marks

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QUESTION 2

You are working as a propulsion engineer analysing the performance of a turbojet engine used to power a jet aircraft during cruise flight **(Figure 2)**. **The aircraft requires a thrust of 30,000 N to maintain a steady cruising speed of 250 m/s**. The engine achieves this thrust by accelerating the incoming air to an exhaust velocity of 800 m/s relative to the aircraft.

Assume steady, one-dimensional flow, neglect pressure thrust and assume that the fuel mass flow rate is negligible compared with the airflow rate.

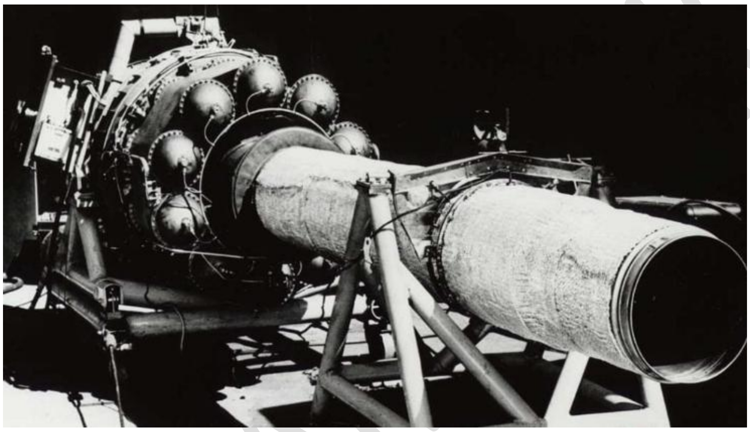

Figure 2 turbojet engine

a) Determine the propulsive efficiency of the engine under the given operating conditions. Clearly show the formula used and state any assumptions made.

[10 Marks]

b) Using the momentum principle, calculate the mass flow rate of air required to produce the specified thrust.

[10 Marks]

c) If the jet engine develops a propulsive power of 6 MW and the total energy input rate from fuel combustion is 15 MW, calculate the thermal efficiency of the engine.

[5 Marks]

Total 25 marks

QUESTION 3

You are a mechanical engineer working on the design and performance optimisation of air-cooling fans and heat exchangers shown in **Figure 3**. The fan blades and flow channels are designed using aerofoil-shaped profiles to enhance airflow efficiency and reduce energy consumption. A thorough understanding of aerodynamic forces is essential for improving the overall thermal and mechanical performance of the system.

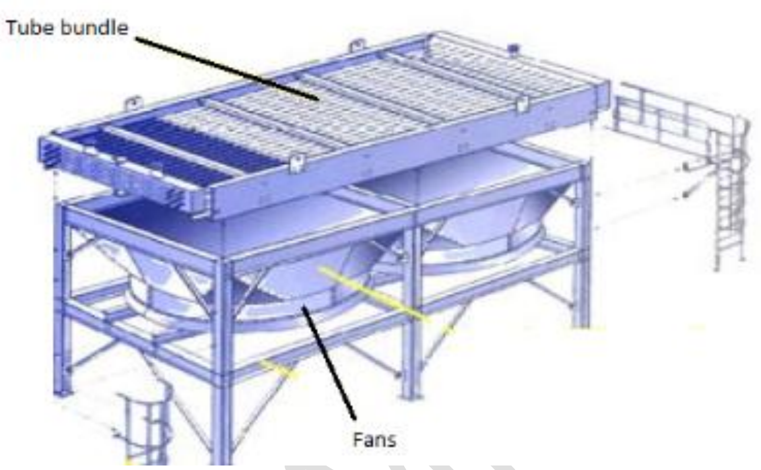

Figure 3 Air-cooling fans and heat exchangers

a) Explain clearly the concepts of drag force and drag coefficient, as well as lift force and lift coefficient, and discuss their importance in aerodynamic design and performance evaluation of aerofoil-based components.

[10 Marks]

b) With the help of a clearly labelled diagram, describe the key components and geometric features of an aerofoil, such as the leading edge, trailing edge, chord line, camber, and thickness.

[5 Marks]

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Question 3 continued

c) Discuss how variations in the angle of attack influence aerodynamic parameters like lift, drag, and flow separation, and explain what happens when the angle exceeds the critical value.

[5 Marks]

d) Provide practical examples of how the aerofoil concept is applied in cooling fans, condensers, or air-cooled heat exchangers within thermal power systems, and explain how these designs contribute to improved airflow, reduced power losses, and enhanced overall plant efficiency.

[5 Marks]

Total 25 marks

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QUESTION 4

A **two-degree-of-freedom active suspension system** is shown in Figure 4. The sprung mass m_s=350 kg, unsprung mass m_u = 40kg, suspension stiffness K_s = 22,000N/m, tryre stiffness K_t = 180000N/m, and damping coefficient c_s = 1200Ns/m. The system is subject to a road disturbance input r(t) and an active control force $F_a(t)$.

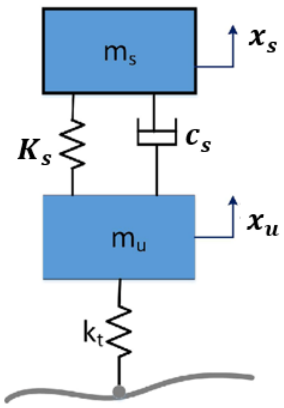

Figure 4 Two-degree-of-freedom active suspension system

a) Derive the **coupled differential equations** governing the motion of the sprung and unsprung masses.

[5 Marks]

b) Obtain the **state-space representation** of the system with input $F_a(t)$ and disturbance r(t).

[8 Marks]

c) Determine the **transfer function** between the control input $F_a(t)$ and the sprung mass displacement $x_s(t)$.

[6 Marks]

d) Assess the **stability, controllability, and observability** of the system. (6marks)

[6 Marks]

Total 25 marks

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QUESTION 5

A non-minimum phase industrial plant is represented by the transfer function shown in **Figure 5**. A unity feedback control system is to be designed using a lead-lag compensator.

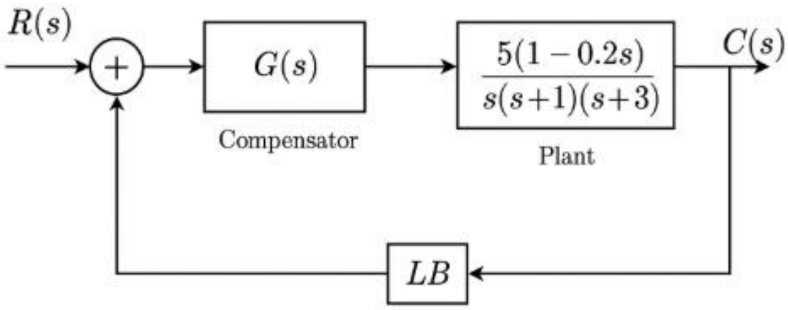

Figure 5 Non-minimum phase industrial plant

a) Evaluate the open-loop performance of the system by determining its steady-state error, phase margin, and gain margin.

[8 Marks]

b) Design a lead compensator that achieves a phase margin of at least 45° and a steady-state error less than 5% for a unit ramp input.

[7 Marks]

c) Introduce a lag element to further reduce steady-state error without significantly affecting phase margin.

[6 Marks]

d) Discuss how the design meets IMechE professional competencies related to control system performance, safety, and sustainability

[4 Marks]

Total 25 marks

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QUESTION 6

A digital control system regulates a thermal plant with a continuous transfer function below. The plant is controlled via a digital PI controller implemented with a sampling time Ts =0.2s

$$G_{PS} = \frac{2}{s(0.5s + 1}$$

a) Derive the discrete equivalent transfer function) $G_p(z)$ using the zero-order hold (ZOH) method.

[7 Marks]

b) Design a PI controller in the z-domain such that the dominant closed-loop poles yield a settling time less than 1.5 seconds and overshoot below 10%.

[8 Marks]

c) The controller is implemented on a 12-bit DAC with an input voltage range of 0–10 V. Calculate the DAC resolution in volts. Discuss how quantisation and sampling effects influence control performance and system stability at high frequencies.

[10 Marks]

Total 25 marks

END OF QUESTIONS

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Formula Sheet

Blocks with feedback loop

G(s) = $\dfrac{Go(s)}{1+Go(s)H(s)}$ (for a negative feedback)

G(s) = $\dfrac{Go(s)}{1-Go(s)H(s)}$ (for a positive feedback)

Steady-State Errors

$e_{ss} = \lim\limits_{s \to 0}[s(1-G_O(s))\theta_i(s)]$ (for an open-loop system)

$e_{ss} = \lim\limits_{s \to 0}[s\dfrac{1}{1+G_o(s)}\theta_i(s)]$ (for the closed-loop system with a unity feedback)

$e_{ss} = \lim\limits_{s \to 0}[s\dfrac{1}{1+\dfrac{G_0(s)}{1+G_0(s)[H(s)-1]}}\theta_i(s)]$ (if the feedback H(s) ≠ 1)

$e_{ss} = \lim\limits_{s \to 0}[-s \cdot \dfrac{G_2(s)}{1+G_2(s)G_1(s)} \cdot \theta_d]$ (if the system subjects to a disturbance input)

$$\text{state} - \text{space matrices for a second} - \text{order system } \frac{bo}{(s^2 + a1s + ao)}$$

$$\frac{{\omega_n}^2}{s^2 + 2\beth\omega_n s + {\omega_n}^2}$$

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Formula sheet continued

Laplace Transforms

A unit impulse function 1

A unit step function $\frac{1}{s}$

A unit ramp function $\frac{1}{s^2}$

First order Systems

$$G(s) = \frac{\theta_o}{\theta_i} = \frac{G_{ss}(s)}{\tau s + 1}$$

$$\tau\left(\frac{d\theta_o}{dt}\right) + \theta_o = G_{ss}\theta_i$$

$\theta_O = G_{ss}(1 - e^{-t/\tau})$ (for a unit step input)

$\theta_O = AG_{ss}(1 - e^{-t/\tau})$ (for a step input with size A)

$\theta_o(t) = G_{ss}(\frac{1}{\tau})e^{-(t/\tau)}$ (for an impulse input)

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Formula sheet continued

Second-order systems

$$\frac{d^2\theta_o}{dt^2} + 2\zeta\omega_n \frac{d\theta_o}{dt} + \omega_n^2\theta_o = b_o\omega_n^2\theta_i$$

$$G(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{b_o\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_d t_r = 1/2\pi$ $\quad$ $\omega_d t_p = \pi$

P.O. = $\exp\left(\frac{-\zeta\pi}{\sqrt{(1-\zeta^2)}}\right) \times 100\%$

$t_s = \frac{4}{\zeta\omega_n}$ $\quad$ $\omega_d = \omega_n\sqrt{(1-\zeta^2)}$

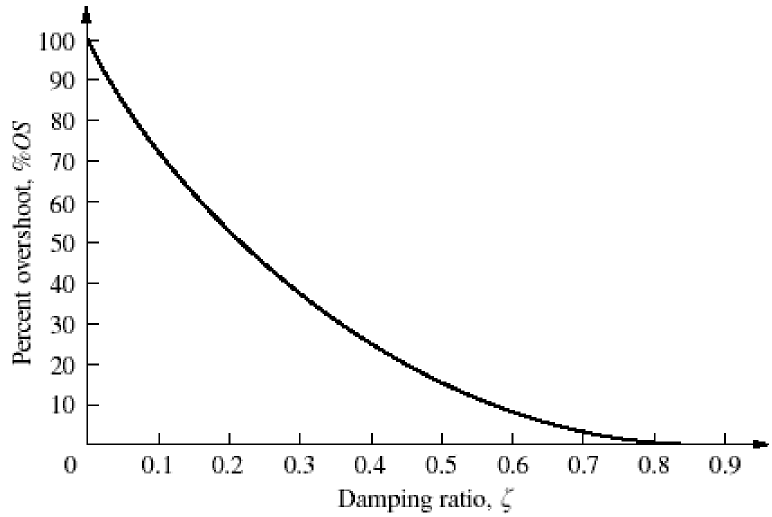

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Table 4.1 Laplace transforms

Laplace transform	*Time function*	*Description of time function*
1		A unit impulse
$\frac{1}{s}$		A unit step function
$\frac{e^{-sT}}{s}$		A delayed unit step function
$\frac{1-e^{-sT}}{s}$		A rectangular pulse of duration T
$\frac{1}{s^2}$	t	A unit slope ramp function
$\frac{1}{s^3}$	$\frac{t^2}{2}$	
$\frac{1}{s+a}$	e^{-at}	Exponential decay
$\frac{1}{(s+a)^2}$	$t e^{-at}$	
$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	
$\frac{a}{s(s+a)}$	$1-e^{-at}$	Exponential growth
$\frac{a}{s^2(s+a)}$	$t-\frac{(1-e^{-at})}{a}$	
$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}-ate^{-at}$	
$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}$	
$\frac{ab}{s(s+a)(s+b)}$	$1-\frac{b}{b-a}e^{-at}+\frac{a}{b-a}e^{-bt}$	
$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(c-a)(a-b)}+\frac{e^{-ct}}{(a-c)(b-c)}$	
$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	Sine wave
$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	Cosine wave
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	Damped sine wave
$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	Damped cosine wave
$\frac{\omega^2}{s(s^2+\omega^2)}$	$1-\cos \omega t$	
$\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}$	$\frac{\omega}{\sqrt{(1-\zeta^2)}}e^{-\zeta\omega t}\sin[\omega\sqrt{(1-\zeta^2)}t]$	
$\frac{\omega^2}{s(s^2+2\zeta\omega s+\omega^2)}$ with $\zeta < 1$	$1-\frac{1}{\sqrt{(1-\zeta^2)}}e^{-\zeta\omega t}\sin[\omega\sqrt{(1-\zeta^2)}t+\phi]$ with $\zeta=\cos\phi$	

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Table 15.1 z-transforms

Sampled $f(t)$, sampling period T	$F(z)$
Unit impulse, $\delta(t)$	1
Unit impulse delayed by kT	z^{-k}
Unit step, $u(t)$	$\frac{z}{z-1}$
Unit step delayed by kT	$\frac{z}{z^k(z-1)}$
Unit ramp, t	$\frac{Tz}{(z-1)^2}$
t^2	$\frac{T^2 z(z+1)}{(z-1)^3}$
e^{-at}	$\frac{z}{z-e^{-aT}}$
$1-e^{-at}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
$t\,e^{-at}$	$\frac{Tz\,e^{-aT}}{(z-e^{-aT})^2}$
$e^{-at}-e^{-bt}$	$\frac{(e^{-aT}-e^{-bT})z}{(z-e^{-aT})(z-e^{-bT})}$
$\sin\omega t$	$\frac{z\sin\omega T}{z^2-2z\cos\omega T+1}$
$\cos\omega t$	$\frac{z(z-\cos\omega T)}{z^2-2z\cos\omega T+1}$
$e^{-at}\sin\omega t$	$\frac{z\,e^{-aT}\sin\omega T}{z^2-2z\,e^{-aT}\cos\omega T+e^{-2}}$
$e^{-at}\cos\omega t$	$\frac{z(z-e^{-aT}\cos\omega T)}{z^2-2z\,e^{-aT}\cos\omega T+e^{-2}}$

Table 15.2 z-transforms

$f[k]$	$f[0], f[1], f[2], f[3], \ldots$	$F(z)$
$1u[k]$	$1, 1, 1, 1, \ldots$	$\frac{z}{z-1}$
a^k	$a^0, a^1, a^2, a^3, \ldots$	$\frac{z}{z-a}$
k	$0, 1, 2, 3, \ldots$	$\frac{z}{(z-1)^2}$
ka^k	$0, a^1, 2a^2, 3a^3, \ldots$	$\frac{az}{(z-a)^2}$
ka^{k-1}	$0, a^0, 2a^1, 3a^2, \ldots$	$\frac{z^2}{(z-a)^2}$
e^{-ak}	$e^0, e^{-a}, e^{-2a}, e^{-3a}, \ldots$	$\frac{z}{z-e^{-a}}$

$$F = \dot{m} \times (V_{\text{exit}} - V_{\text{jet}})$$

$$\eta_{\text{propulsive}} = \frac{2v_0}{v_0 + v_e}$$

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