UNIVERSITY OF GREATER MANCHESTER

OFF CAMPUS DIVISION

WESTERN INTERNATIONAL COLLEGE, RAS AL KHAIMAH

BENG(HONS) ELECTRICAL AND ELECTRONIC ENGINEERING

SEMESTER ONE EXAMINATION 2025/26

ENGINEERING ELECTROMAGNETISM

MODULE NO: EEE6012

Date: Saturday, 17th January 2025 **Time: 10:00am – 12:30pm**

INSTRUCTIONS TO CANDIDATES:

There are FIVE (5) questions on this paper.

Answer any FOUR (4) questions.

All questions carry equal marks.

Q1. To validate a rectangular sensor prototype that operates in air, you must evaluate electric field quantities in its operating region.

$$V = x^2y + z^2 \quad volts$$

(i) Explain Maxwell's equations and their physical significance. Using the given potential function, evaluate the electric field intensity(E) and electric flux density D.

[11 marks]

(ii) Analyse the volume charge density ρ_v and the total charge enclosed(Q) within the rectangular region $0 \le x \le 1\,, 0 \le y \le 2\,, 0 \le z \le 1\,m$

[10 marks]

(iii) Assess the work done in moving a 5 µC charge from point A (1, 2, 1) to B (0,1,1).

[4 marks]

(Total 25 marks)

Q2. a) Given the magnetic vector potential in a certain region in a space as

$$A \ = (4\rho z)a_{\varphi} + 2\rho^2 a_z \, \mathrm{Wb/m}$$

determine the magnetic vector potential(A), magnetic flux density (B), magnetic field intensity(H) and current density(J) at point P (2, -1,3).

[15 marks]

b) A toroid whose dimensions are shown in Figure Q2 has N turns and carries current I.

(i) Analyse the magnetic field intensity(H) inside and outside the toroid.

[5 marks]

Question 2 continued over the page….

Q2 continued…

(ii) Given the toroidal core has $\rho_0 = 10\ cm$ and a circular cross section with a = 1 cm. If the core is made of Steel ($\mu = 100\mu_0$) and has a coil with 200 turns, determine the amount of current that will produce a flux of 0.5 mWb in the core.

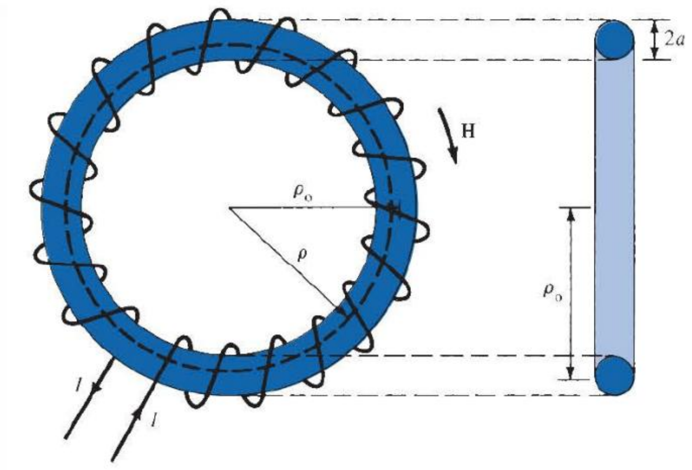

Figure Q2 a toroid with a circular cross section.

[5 marks]

(Total 25 marks)

Q3. a) In a lossless dielectric $H = 0.1\,cos(\omega t - z)a_x + 0.5\sin(\omega t - z)\,a_y$ A/m and $\eta = 60\pi$, $\mu_r = 1$. Evaluate ε_r, ω and E.

[12 marks]

b) Define the Poynting vector and explain its role in representing power flow in electromagnetic waves. Using the field expression $H = 0.2\,cos(\omega t - \beta x)a_z$ A/m, analyse the corresponding Poynting vector.

[13 marks]

(Total 25 marks)

Please turn the page

Q4.

a) A transmission line is terminated by a resistive load which is less than the characteristic impedance and the voltage standing wave ratio (VSWR) on the line is 2.0. Evaluate,

i) the voltage reflection coefficient of the load. **[3 marks]**

ii) the ratio of the load impedance to the characteristic impedance of the line. **[3 marks]**

iii) the ratio of the reflected wave to the incident wave (express in decibels). **[3 marks]**

b) A coaxial cable with a characteristic impedance of 50 Ω is insulated with a dielectric whose relative permittivity is 2.7. Given that a 1 metre length of this cable is terminated in a 75 Ω resistor, analyse,

i) the wavelength of signals on the line at 500 MHz and 1 GHz. **[8 marks]**

ii) the input impedance of the cable at frequencies 500 MHz and 1 GHz. **[8 marks]**

(Total 25 Marks)

Q5.

a) The transmitting and receiving antennas are separated by a distance of 200λ and have directive gains of 25 dB and 18 dB, respectively. If 5 MW of power is to be received, analyse the minimum transmitted power. **[9 marks]**

Q5 continued over the page….

Q5 continued...

b) An antenna in air radiates a total power of 100 kW so that a maximum radiated electric field strength of 12 mV/m is measured 20 km from the antenna. Evaluate,

i) Antenna directivity in dB.

[7 marks]

ii) Antenna maximum power gain if radiation efficiency h_r = 98%.

[2 marks]

c) Analyse the maximum effective area of a $\lambda/2$ wire dipole operating at 30 MHz. Also evaluate how much power is received with an incident plane wave of strength 2 mV/m.

[7 marks]

(Total 25 Marks)

End of Questions

PLEASE TURN THE PAGE FOR EQUATION SHEETS

EQUATION SHEETS

CIRCULAR CYLINDRICAL COORDINATES (ρ, ϕ, z)

$$\boxed{\rho = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\frac{y}{x}, \quad z = z}$$

SPHERICAL COORDINATES (r, θ, ϕ)

$$\boxed{r = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}, \quad \phi = \tan^{-1}\frac{y}{x}}$$

$$\boxed{x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta}$$

DIFFERENTIAL LENGTH, AREA, AND VOLUME

A. Cartesian Coordinate Systems

1. Differential displacement is given by

$$\boxed{d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z}$$

2. Differential normal surface area is given by

$$\boxed{\begin{aligned} d\mathbf{S} = {} & dy\,dz\,\mathbf{a}_x \\ & dx\,dz\,\mathbf{a}_y \\ & dx\,dy\,\mathbf{a}_z \end{aligned}}$$

3. Differential volume is given by

$$\boxed{dv = dx\,dy\,dz}$$

B. Cylindrical Coordinate Systems

1. Differential displacement is given by

$$\boxed{d\mathbf{l} = d\rho\,\mathbf{a}_\rho + \rho\,d\phi\,\mathbf{a}_\phi + dz\,\mathbf{a}_z}$$

2. Differential normal surface area is given by

$$\boxed{\begin{aligned} d\mathbf{S} = {} & \rho\,d\phi\,dz\,\mathbf{a}_\rho \\ & d\rho\,dz\,\mathbf{a}_\phi \\ & \rho\,d\rho\,d\phi\,\mathbf{a}_z \end{aligned}}$$

and illustrated in Figure 3.4.

3. Differential volume is given by

$$\boxed{dv = \rho\,d\rho\,d\phi\,dz}$$

DEL OPERATOR

$$\boxed{\nabla = \frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z}$$

$$\boxed{\nabla = \mathbf{a}_\rho\frac{\partial}{\partial \rho} + \mathbf{a}_\phi\frac{1}{\rho}\frac{\partial}{\partial \phi} + \mathbf{a}_z\frac{\partial}{\partial z}}$$

$$\boxed{\nabla = \mathbf{a}_r\frac{\partial}{\partial r} + \mathbf{a}_\theta\frac{1}{r}\frac{\partial}{\partial \theta} + \mathbf{a}_\phi\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}}$$

GRADIENT OF A SCALAR

$$\boxed{\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z}$$

$$\boxed{\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z}$$

$$\boxed{\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi}$$

DIVERGENCE OF A VECTOR

$$\boxed{\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

$$\boxed{\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}}$$

$$\boxed{\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \mathbf{A}_\phi}{\partial \phi}}$$

CURL OF A VECTOR

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\,\mathbf{a}_\phi & \mathbf{a}_z \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \mathbf{a}_r & r\,\mathbf{a}_\theta & r\sin\theta\,\mathbf{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{vmatrix}$$

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{A}\, dv = 0$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$$

$$\boxed{\mathbf{E} = \frac{Q}{4\pi\varepsilon_o r^2}\,\mathbf{a}_r}$$

$$Q = \int_L \rho_L \, dl \qquad \text{for line charge}$$

$$Q = \int_S \rho_S \, dS \qquad \text{for surface charge}$$

$$Q = \int_v \rho_v \, dv \qquad \text{for volume charge}$$

ELECTRIC FLUX DENSITY

$$\boxed{\mathbf{D} = \varepsilon_o \mathbf{E}}$$

$$Q = \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$$

$$\boxed{\rho_v = \nabla \cdot \mathbf{D}}$$

$$\boxed{\mathbf{E} = -\nabla V}$$

electric flux through a surface S is

$$\Psi = \int_S \mathbf{D} \cdot d\mathbf{S}$$

$$I = \oint \mathbf{J} \cdot d\mathbf{S} = \int \nabla \cdot \mathbf{J} \, dv$$

$$\boxed{\mathbf{J} = \sigma \mathbf{E}}$$

$$\rho_v = ne$$

$$J = \sigma E$$

$$\begin{aligned} W &= -Q\int_A^B \mathbf{E}\cdot d\mathbf{l} = QV_{AB} \\ &= Q(V_B - V_A) \end{aligned}$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$$

$$\mathrm{F} = \int_v \rho_v \mathrm{E}\, dv$$

$$\boxed{\oint_L \mathbf{H}\cdot d\mathbf{l} = I_{enc}}$$

$$\boxed{\nabla \times \mathbf{H} = \mathbf{J}}$$

$$\boxed{\mathbf{B} = \mu_o \mathbf{H}}$$

$$\boxed{\mu_o = 4\pi \times 10^{-7}\ \mathrm{H/m}}$$

$$\boxed{\Psi = \int_S \mathbf{B}\cdot d\mathbf{S}}$$

$$\boxed{\oint_S \mathbf{B}\cdot d\mathbf{S} = 0}$$

$$\oint_S \mathbf{B}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{B}\, dv = 0$$

$$\mathrm{B} = \mu_r\, \mu_0\, (N/2\pi r)\ i$$

$$\mathrm{H}_\phi = \left[\begin{array}{ll} \dfrac{I\rho}{2\pi a^2}, & \rho < a \\[2ex] \dfrac{I}{2\pi\rho}, & \rho > a \end{array}\right.$$

$$\nabla \cdot \mathbf{D} = \rho_v \qquad \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\nabla \times \mathbf{E} = \mathbf{0} \qquad \oint_L \mathbf{E} \cdot d\mathbf{l} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad \oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A}}$$

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\boxed{\nabla \cdot \mathbf{A} = 0}$$

$$\boxed{\mathbf{F} = \oint_L I \, d\mathbf{l} \times \mathbf{B}}$$

$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o\varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r}$$

$$\boxed{\mathcal{P} = \mathbf{E} \times \mathbf{H}}$$

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$k = \beta = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\mathcal{P}_{\text{ave}} = \frac{1}{2}\text{Re}\left(\mathbf{E}_s \times \mathbf{H}_s^*\right) = \frac{E_o^2}{2\eta}\mathbf{a}_k$$

$$P_{\text{ave}} = \int \mathcal{P}_{\text{ave}} \cdot d\mathbf{S} = \mathcal{P}_{\text{ave}} \cdot S\,\mathbf{a}_n$$

TRANSMISSION LINES

1 Np= 8.686db

Propagation constant

$$\gamma = \alpha + j\beta$$

Wave velocity, $u = \frac{\omega}{\beta} = f\lambda$

Wavelength, $\lambda = \frac{2\pi}{\beta}$

Input impedance

$$Z_{in} = Z_o \left[\frac{Z_L + Z_0\, tanh tanh\, \gamma l}{Z_0 + Z_L\, tanh tanh\, \gamma l} \right]$$

$$tanh\ _{tanh}\left(x \pm jy\right) = \frac{sinh sinh\, 2x}{cosh cosh\, 2x + cos cos\, 2y} \pm j\, \frac{sinh sinh\, 2y}{cosh cosh\, 2x + cos cos\, 2y}$$

$$Z_{in} = \frac{V_s\ (z)}{I_s(z)} = \frac{Z_0(V_0^+ + V_0^-)}{V_0^+ - V_0^-}$$

Voltage and current at any point z

$$V_s(z) = V_0^+\, e^{-\gamma z} + V_0^-\, e^{\gamma z}$$

$$I_s(z) = \frac{V_0^+}{Z_0}\, e^{-\gamma z} - \frac{V_0^-}{Z_0}\, e^{\gamma z}$$

$$V_0^+ = \frac{1}{2}\left(V_0 + Z_0 I_o\right)$$

$$V_0^- = \frac{1}{2}\left(V_0 - Z_0 I_o\right)$$

Sending end current and voltage

$$I_0 = \frac{V_g}{Z_{in} + Z_g}$$

$$V_0 = Z_{in} I_0 = \frac{Z_{in}}{Z_{in} + Z_g}\, V_g$$

Reflection coefficient

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Standing wave ratio

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

Antenna

Wavelength

$$\lambda = \frac{c}{f}$$

Power radiated,

$P_{rad} or\ W = I_{rms}^2 \text{ x } R_{rad}$

Effective area,

$A_e = \frac{\lambda^2}{4\pi} \text{D}$

Capture area of a circular aperture,

$$A_e = \frac{\pi D^2}{4}$$

Radiation Efficiency

$\eta = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_l}$

$\eta_r = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_l}$

Directivity

$\text{D} = \frac{4\pi\ U_{max}}{P_{rad}}$

U_{max} – Radiation intensity

$$D = \frac{4\pi}{\lambda^2} A_e$$

Gain of an Antenna

$G = \boldsymbol{\eta} D$

η – Radiation Efficiency

$G = KD$

$$G = K \frac{4\pi}{\lambda^2} A_e$$

K- antenna factor , 1 if no losses present

Gain in db, $G_{db} = 10\, log_{10} G$

Q factor

$Q = \frac{f_r}{\Delta f}$

Δf *- Bandwidth*

END OF PAPER